

# Non-forward Balitsky-Kovchegov equation and Vector Mesons

Robi Peschanski<sup>1</sup>, Cyrille Marquet<sup>2</sup> and Gregory Soyez<sup>3</sup>

1- Service de Physique Théorique - CEA/Saclay, 91191 Gif-sur-Yvette Cedex, FRANCE

2- RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

3- Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Considering the Balitsky-Kovchegov QCD evolution equation in full momentum space, we derive the travelling wave solutions expressing the nonlinear saturation constraints on the dipole scattering amplitude at non-zero momentum transfer. A phenomenological application to elastic vector meson production shows the compatibility of data with the QCD prediction: an *enhanced* saturation scale at intermediate momentum transfer.

## 1 Motivation

The saturation of parton densities at high energy has been mainly studied for the forward dipole-target scattering amplitude  $\mathcal{T}(r, q = 0, Y)$ , where  $r, q, Y$  are, respectively, the dipole size, the momentum transfer and the total rapidity of the process. For instance, the corresponding QCD Balitsky-Kovchegov (BK) equation [2] has been shown to provide a theoretical insight on the “geometric scaling” properties [3] of the related  $\gamma^*$ -proton cross-sections. Indeed, it can be related to the existence of a scaling for  $\mathcal{T}(r, q = 0, Y) \sim \mathcal{T}(r^2 Q^2(Y))$  where the saturation scale is  $Q^2(Y) \sim \exp cY$  and the constant  $c$  can be interpreted as the critical speed of “travelling wave” solutions of the nonlinear BK equation [4]. Our theoretical and phenomenological subjects are the extension of these properties to the non-forward amplitude  $\mathcal{T}(r, q \neq 0, Y)$ , which is phenomenologically relevant for the elastic production of vector mesons in deep inelastic scattering.

## 2 BK equation in full momentum space

In order to study the properties of  $\mathcal{T}(r, q \neq 0, Y)$ , one has first to deal with both conceptual and technical difficulties. It is known that the BK formalism has been originally derived in impact parameter  $b$  but then its validity especially at large  $b$  is questionable, since it leads to non physical power-law tails. Hence we start with the formulation of the BK equation in momentum  $q$ , which is more *local* but has a non-trivial nonlinear form [5]. In fact, despite this problem, the general method of travelling wave solutions can be extended in the non-forward domain [6]. It consists in 3 steps: first, one solves the equation restricted to its linear

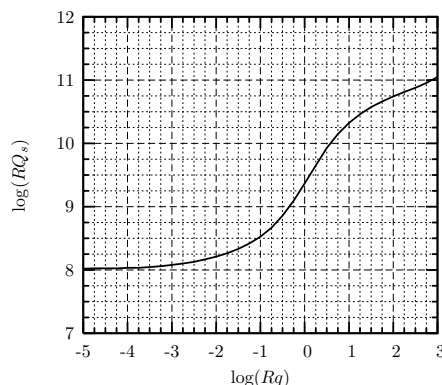


Figure 1:  $q^2$ -dependent saturation scale

part which is related to the non-forward Balitsky Fadin Kuraev Lipatov (BFKL) equation [7] for the dipole-dipole amplitude *via* factorisation and whose solution takes the form of a linear superposition of waves. Second, one finds that the nonlinearities act by selecting the travelling wave with *critical* speed  $c$ , in a way which, interestingly, is independent of the specific structure of the nonlinear damping terms. Third, one obtains after enough rapidity evolution, a solution which appears independent from initial conditions ( $\mathcal{T}_0 \sim r^{2\gamma_0}$ ), provided these are sharper than the critical travelling wave front profile  $\mathcal{T} \sim r^{2\gamma_c}$ , with  $\gamma_0 > \gamma_c$ . Interestingly enough, QCD color transparency satisfies this criterium. Applying these gen-

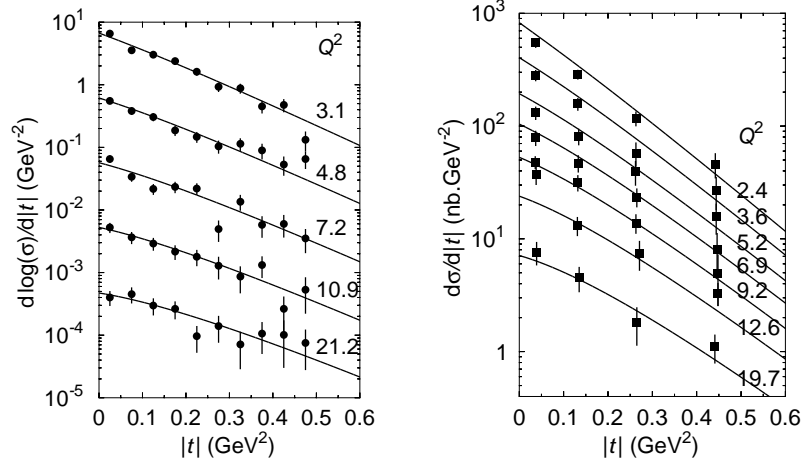


Figure 2:  $\rho$  (H1) and  $\phi$  (ZEUS) differential cross-sections at  $W = 75$  GeV

eral results on the non-forward case one finds the following QCD predictions, depending on the relative magnitude of three scales involved in the process, namely  $q$ ,  $k_T^{-1}$  (the target size) and  $k_P^{-1} \equiv r$  (the projectile *i.e.* dipole size).

- Near-Forward region  $q \ll k_T \ll k_P$  :  $Q_s^2(Y) \sim k_T^2 \exp cY$
- Intermediate transfer region  $k_T \ll q \ll k_P$  :  $Q_s^2(Y) \sim q^2 \exp cY$
- High transfer region  $q \ll k_T \ll k_P$  : No saturation.

Our main prediction is thus the validity of the forward travelling wave solution extended in the non-forward intermediate-transfer domain but with an *enhanced* saturation scale by the ratio  $q^2/k_T^2$ , where  $k_T$  is a typically small, nonperturbative scale. Hence we are led to predict *geometric scaling* properties with a purely perturbative initial saturation scale given by the transverse momentum. This saturation scale *enhancement* prediction is confirmed by numerical simulations of the BK solutions as shown in Fig.1.

### 3 QCD Saturation Model for Exclusive VM production

The differential cross-section for exclusive vector meson (VM) production at HERA, see Fig.2, can be theoretically obtained from the non-forward dipole-proton amplitude and from  $\Phi_{T,L}^{\gamma^*V}$ , the overlap functions between the (longitudinal and transverse) virtual photon and

vector meson wave-functions [8]. For completion, we used two different VM wave-functions of the literature, without noticeable difference in our conclusions. One writes

$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow V p}}{dq^2} = \frac{1}{16\pi} \left| \int d^2r \int_0^1 dz \Phi_{T,L}^{\gamma^* V}(z, r; Q^2, M_V^2) e^{-izq \cdot r} \mathcal{T}(r, q, Y) \right|^2,$$

Following theoretical prescriptions, we consider a forward dipole-proton amplitude  $\mathcal{N}_{IIM}$  satisfactorily describing the total DIS cross-sections in a saturation model [9]. We just make the saturation scale varying with  $q^2$ , following the trend shown in Fig.1 and starting from the forward model one  $Q_s^2(Y)$ , one writes

$$T(r, q; Y) = 2\pi R_p^2 e^{-Bq^2} \mathcal{N}_{IIM}(r^2 Q_s^2(Y, q)) ; Q_s^2(q, Y) = Q_s^2(Y) (1 + c q^2) .$$

Cross-sections	$q^2$ -Sat.	fixed-Sat.
$\rho, \sigma_{\text{el}}$	1.156	1.732
$\rho, \frac{d\sigma}{dt}$	1.382	1.489
$\phi, \sigma_{\text{el}}$	1.322	2.247
$\phi, \frac{d\sigma}{dt}$	1.076	0.931
Total	1.212	1.480

Table 1: Comparison of the  $\chi^2/\text{points}$

Table compares the saturation fits for fixed and  $q^2$ -dependent scales, with a favour for the enhanced-scale model in the total. The model gives a comparable fit with a more conventional non-saturation model using a  $Q^2$ -dependent slope  $B \propto M_V^2 + Q^2$ . Some of our results for the cross-sections are displayed in the figures. In Fig.2, one shows the results of the fit for  $\rho$ -production (H1) and  $\phi$ -production (ZEUS) differential cross-sections for a total  $\gamma^* - p$  energy  $W = 75\text{GeV}$  and different  $Q^2$  values. Let us finally present our predictions for the

The factor  $2\pi R_p^2 e^{-Bq^2}$  comes from the non-perturbative proton form factor. For clarity of the analysis, we considered only  $B$  and  $c$  as free parameters of the non-forward parametrisations, the others being independently fixed by the forward analysis.

In Table 1, one displays the  $\chi^2/\text{point}$  obtained by a fit of  $\rho$  (47 data points) and  $\phi$  (34 points) total elastic production cross-sections and of  $\rho$  (50 data points) and  $\phi$  (70 points) differential cross-sections. The Ta-

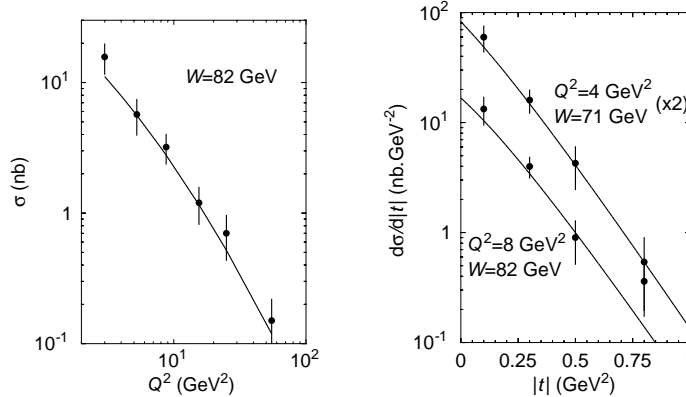


Figure 3: Predictions for the DVCS measurements. Left plot: cross-section, right plot: differential cross-section.

DVCS cross-section, which is obtained without any free parameter from our analysis. In Fig.3, they are compared with the available data and the agreement is good in the simple chosen parametrisation.

## 4 Conclusions

Let us summarize our new results

- *Saturation at non-zero transfer:* The Balitsky-Kovchegov QCD evolution equation involving full momentum transfer predicts (besides the known  $q = 0$  case) saturation in the *intermediate* transfer range, namely for  $Q_0 < q < Q$ , where  $Q_0$  (resp.  $Q$ ) is the target (resp. projectile) typical scale.

- *Characterisation of the universality class:* The universality class of the corresponding travelling-wave solutions is governed by a purely perturbative saturation scale  $Q_s(Y) \equiv q^2 \Omega(Y)$ , where  $\Omega(Y) \sim e^{cY}$  is the same rapidity evolution factor as in the forward case. Consequently the *intermediate transfer* saturation scale gets enhanced by a factor  $q^2/Q_0^2$ .

- *Phenomenology of Vector mesons:* The QCD predictions are applied in the experimentally accessible *intermediate transfer* range of vector meson production. The model uses an interpolation between the forward and non-forward saturation scale together with a parameter-frozen forward saturation model. It fits better the data on  $\rho$  (H1) and  $\phi$  (ZEUS) cross-sections than for a non-enhanced saturation.

- *Prospects:* The next phenomenological prospect is to add charm to the discussion, both with the modification of the forward case by including the charm contribution [10] and by also considering the production of  $\Psi$  mesons. On a theoretical ground, it would be interesting to go beyond the mean-field approximation of the BK equation.

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